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Results on the Grundy chromatic number of graphs. (English summary)

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This paper contains several interesting results on the Grundy numbers of finite simple graphs. The Grundy number $\Gamma(G)$ of such a graph is the greatest number of colors needed to color the vertices in a "first-fit" or "on-line" coloring, in which the vertices are colored in some order by the rule that each vertex in its turn is colored with the smallest positive integer not appearing among its neighbors that have already been colored.

In Section 2 it is shown that for fixed k it can be decided in no more than $c(k)n^{(2^k)}$ steps whether or not $\Gamma(G) > k$, where $n = n(G)$ is the order of G and $c(k)$ depends only on k. This result does not conflict, of course, with a result of N. Goyal and S. Vishvanathan from 1997, still unpublished according to the references, that determining $\Gamma(G)$ is an NP-hard problem.

In Section 3 a number of results are given on the Grundy numbers of the complements of bipartite graphs, most notably that if G is the complement of H, bipartite, then $\Gamma(G) = n - m$, where $n = n(G)$ and m is the edge domination number of H, meaning the domination number of the line graph of H. It follows from results of M. Yannakakis and F. Gavril [SIAM J. Appl. Math. **38** (1980), no. 3, 364–372; MR0579424 (83c:68086)] that determining the Grundy numbers of the complements even of bipartite graphs with maximum degree 3 is an NP-hard problem, which trumps the result of Goyal and Vishvanathan mentioned above.

In Section 4 some Nordhaus-Gaddum-type inequalities for certain classes of graphs (nearly regular graphs, forests, bipartite graphs with conditions on the maximum degree) are proved, leading to the conjecture that for any graph on n vertices, the sum of the Grundy numbers of the graph and its complement is no greater than $n + 2$. It is revealed in a note added in proof that this conjecture has been disproven by Z. Füredi, A. Gyárfás, G. N. Sárközy, and S. Selkow ["Inequalities for the first-fit chromatic number", J. Graph Theory, to appear].

In Section 5 the subject of graphs whose Grundy number equals their chromatic number, called well-colored graphs, is broached, and it is shown that for each non-negative integer r the recognition of graphs whose Grundy number does not exceed their chromatic number by more than r is a coNP-complete problem. Recognition of well-colored graphs—the case $r = 0$—is therefore coNP-complete.

Reviewed by *Peter D. Johnson, Jr.*

References

1. Y. Caro, A. Sebö, M. Tarsi, Recognizing greedy structures, J. Algorithms 20 (1996) 137–156. MR1368720 (96j:68078)
2. C.A. Christen, S.M. Selkow, Some perfect coloring properties of graphs, J. Combin. Theory Ser. B 27 (1979) 49–59. MR0539075 (80k:05055)

3. P. Erdös, W.R. Hare, S.T. Hedetniemi, R. Laskar, On the equality of Grundy and ochromatic number of graphs, J. Graph Theory 11 (1987) 157–159. MR0889347 (88g:05057)
4. N. Goyal, S. Vishvanathan, NP-completeness of undirected Grundy numbering and related problems, Manuscript, Bombay, 1997.
5. A. Gyárfás, Z. Király, J. Lehel, On-line 3-chromatic graphs—II critical graphs, Discrete Math. 177 (1997) 99–122. MR1483441 (99e:05049)
6. A. Gyárfás, J. Lehel, On-line and first-fit coloring of graphs, J. Graph Theory 12 (1988) 217–227. MR0940831 (89c:05037)
7. R. Häggkvist, A.S. Asratian, T.M.J. Denley, Bipartite Graphs and their Applications, Cambridge University Press, Cambridge, 1998. MR1639013 (99e:05121)
8. S.M. Hedetniemi, S.T. Hedetniemi, A. Beyer, A linear algorithm for the Grundy (coloring) number of a tree, Congr. Numer. 36 (1982) 351–363. MR0726079 (85b:68041)
9. D.G. Hoffman, P.D. Johnson Jr., Greedy colorings and the Grundy chromatic number of the n-cube, Bull. ICA 26 (1999) 49–57. MR1683820
10. T.R. Jensen, B. Toft, Graph Coloring Problems, Wiley, New York, 1995. MR1304254 (95h:05067)
11. H.A. Kierstead, W.T. Trotter, On-line graph coloring, in: L.A. McGeoch, D.D. Sleator (Eds.), On-Line Algorithms: Proceedings of DIMACS Workshop, February 11–13, 1991, American Mathematical Society, Providence, RI, 1992, pp. 85–92. MR1165344
12. G.J. Simmons, On the ochromatic number of a graph, Congr. Numer. 40 (1983) 339–366. MR0734382 (85b:05088)
13. J.A. Telle, A. Proskurowski, Algorithms for vertex partitioning problems on partial k-trees, SIAM J. Discrete Math. 10 (1997) 529–550. MR1477655 (99c:68200)
14. M. Yannakakis, F. Gavril, Edge dominating sets in graphs, SIAM J. Appl. Math. 38 (1980) 364–372. MR0579424 (83c:68086)
15. M. Zaker, Grundy chromatic number of the complement of bipartite graphs, Australas. J. Combin. 31 (2005) 325–329. MR2113717 (2005h:05087)
16. Z. Füredi, A. Gyárfás, G. Sárközy, S. Selkow, Inequalities for the First-Fit chromatic number (November 2005).

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.